OPTING-OUT OF PUBLIC EDUCATION IN URBAN ECONOMIES

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ABSTRACT

This paper analyses the question of which households opt out of public education in a multi-community (urban) economy with local school finance and housing markets. In particular, the objective is to investigate whether perfect income stratification across public and private educational sectors predicted by single jurisdiction models and by multiple jurisdiction ones without housing markets holds in this setting. Nechyba (1999) has shown that the existence of a fixed stock of heterogeneous houses can prevent perfect income stratification from arising in equilibrium. Here we demonstrate that, even with homogeneous housing, perfect income stratification is not assured. On the contrary, it is possible to find equilibria in which households from intermediate income intervals use private schools, while richer ones prefer to send their youths to a local public school of higher quality. The paper identifies a new way whereby housing markets affect how the market for education works.
1. INTRODUCTION

The last decade has witnessed the emergence of a hot debate about school finance reform in many countries. Theoretical and empirical research on the economics of education has shed light on key issues within it: the impact of decentralisation on public schools productivity (Hoxby, 1999), the role of different inputs into the production of education (Hanushek, 1997) or the consequences of peer group effects on sorting across schools and neighbourhoods (de Bartolome, 1990, Epple and Romano, 1998, 2002) are only three examples. One fascinating branch of research within this field is the co-existence of private and public education institutions. A voluminous literature on this topic has rapidly developed during this time, partly stimulated by private schools vouchers proposals.

When private alternatives to public education are made available, several questions must be addressed. For instance, it is crucial to know what are the consequences over public and total education investment, or over the distribution of educational benefits across households. An important related question is which households exert choice and opt out of the public system. Clearly, the answer to this question has a significant impact on the others. For example, if students who attend private schools are the brightest ones or those from the richest households, then the cream skimming effect of private education could worsen the situation of poorer and less able students who are left behind in public schools (Epple and Romano, 1998).

The focus of this paper will be on that question, i.e., on how households choose among public and private schools. This issue has been analysed theoretically by Stiglitz (1974), De Fraja (2001) and Epple and Romano (1996), among others. Let us briefly discuss these models. In all of them, agents differ along a single dimension: ability to earn future income in the first two, income in the last one. The public educational sector faces two constraints: public schooling must be free and all students in the public system must receive education of the same quality. Finally, funding levels for public education are decided through majority voting in Stiglitz (1974) and Epple and Romano (1996), and by a welfare maximiser utilitarian government in De Fraja (2001). A pervasive feature of equilibrium in these models is the perfect stratification of households across public and private schools, with the best-off (either the richest or the brightest) choosing (higher quality) private education. When households differ by income, what assures perfect stratification is the normality of educational services. In Stiglitz (1974) and De Fraja (2001), in turn, it is the complementarity of education and ability in the function determining (future) income.

One important exception to the perfect stratification result is in Epple and Romano (1998). In their model, households differ by both income and the student's ability and stratification occurs not only by income but also by ability. Consequently, there are relatively low income households...
with students of relatively high ability which end up using a private school. Because school quality is determined by the average ability of its students, profit maximising private schools make tuition fees a function of ability, in order to internalise the positive externality generated by high ability peers. Hence, even if demand for education does not increase in ability, private schools charge lower tuition fees to high ability students, making the 2-dimensional stratification to arise.

All the models above consider single-jurisdiction economies. This is clearly adequate either if public education is centrally provided or if mobility among jurisdictions can reasonably be ignored, but not if public education is locally provided and mobility across communities is significant as in urban economies. Therefore, choice among public and private schools must also be studied in a multiple-jurisdiction framework with costless mobility.

One distinct feature of these economies is that school choice can be exerted not only among public and private institutions but also among (different quality) local public schools. Because attending the local public school of a particular community requires to reside and pay taxes within it, there exists a link between the quality of public education available for a household and their community of residence. Thus, the choice of residential location becomes a crucial issue, at least, for households choosing public schooling. For those opting-out, in turn, school quality is not linked to the choice of community. However, because public schools quality differentials are likely to be capitalised into housing prices, their choice of community is also affected by the market for education. At the same time, political support for public education falls in the community in which they choose to live. The interactions among the market for education and housing markets are therefore strong in these economies.

Nechyba (1999, 2002) and Bearse et al. (2001) have investigated the workings of decentralised systems of public education with mobility and opting-out. Bearse et al. (2001) develop a quite stylised model with no housing markets in which households differ by income in order to carry-out simulations in a dynamic economy. The objective is to investigate the role of private education when comparing central school finance and a vouchers system to a decentralised regime. In their model, the answer to the question posed above is the same in single and multiple jurisdiction economies: there exists perfect income stratification across educational sectors and the rich households are the ones choosing private schooling. They opt out in order to receive an education of higher quality than that offered in the public sector because educational services are normal.

On the other hand, Nechyba (1999, 2002) develops a rich theoretical model in which each community has a fixed stock of heterogeneous houses, households differ by their endowment of housing wealth and income and by the ability of their student, and peer group effects are an input
for producing school quality. The theoretical model, however, is too complex to yield many analytical results and a computational counterpart is therefore analysed. An important assumption in the analysis in Nechyba (1999) is that student ability and income are perfectly correlated. Under such assumption, again, private schools are always used by high income households (with high ability students). Yet, even in that case, perfect stratification across educational sectors is not assured. In his computational experiments, high quality houses are more abundant in one community, which ends up offering high quality public education. A number of affluent households prefer to reside in that community in order to choose a house of high quality. However, because the local public school is of high quality and the price of housing is very high there, these households decide to continue using public education.

Housing heterogeneity is a first way through which housing markets can prevent perfect income stratification from arising in a multi-community framework. The current paper adopts a theoretical perspective to analyse if perfect stratification across educational sectors maintains in a similar economy with homogeneous housing. In other words, if the rich (and only they) are always the ones who opt out of public education in such setting. The analysis demonstrates that, even with housing homogeneity, housing markets may significantly affect the way households and children sort into communities and schools. In particular, it shows that households from different income intervals may choose private education and that households from the top income interval in the economy may not be among them. That is to say, the analysis concludes that perfect income stratification across public and private education is not assured even in a single-dimensional characteristics space. Moreover, it demonstrates that the best-off households may prefer to send their youths to a public school of higher quality than the best private school in the market. Therefore, "elite" public schools may emerge and survive the competition of private schools.

The paper is organised as follows. The next section introduces the multi-community model with local school finance, opting-out and housing markets. This model is analysed in section 3. Section 4 concludes.

2. THE MODEL

2.1. Communities and housing markets

The economy is composed of a fixed number of communities, J, with exogenous boundaries, which may differ in the amount of land contained within their limits. We adopt a simple specification of the housing market borrowed from Epple and Romano (2002), which turns out to be especially convenient for the purposes of the paper. Houses are homogenous and each household
consumes one (and only one) unit of housing at price $p_h$. Every community $i$ has a backward-L housing supply, horizontal at $c$ (where $c$ is the common construction cost) until community land capacity is reached and vertical at that quantity. Each house requires one lot of land. We assume that the amount of land in the economy is just enough to house the population.

2.2. Education system and taxation

Education is treated as a private good. Educational services are produced from the numeraire, following a technology of production which exhibits constant returns to scale with respect to the number of students and the quantity produced. The cost function $c(x,n)=x·n$ captures this technology. For simplicity sake, it assumes away the influence of peer effects and other inputs such as student ability and effort. This technology of production is assumed to be common for all producers of education.

Every community $i$ may impose a proportional property tax on the value of housing and use the proceeds to provide public educational services $E_i$. Each community chooses the pair $(E_i,t_i)$, where $t$ stands for the tax rate, through a political process, simplified to majority voting. Besides the public system, there exists a private competitive market for education in which households can acquire any amount of educational services at competitive price $p_x=1$.

A couple of notes about the coexistence of public and private schools are necessary. First, as it is usual in models of education, we consider public and private alternatives as mutually exclusive. Therefore, a child cannot receive public and private education simultaneously. Second, while households can acquire as many units of private education as they prefer, regardless of where they live, they can only send their children to the local public school in a given community if they reside and pay taxes in it. This residence requirement that characterises the public sector is crucial for the analysis, as will become evident bellow.

2.3. Households and preferences

The economy is inhabited by a continuum of households, each composed by one adult, the decision-maker, and one school-aged children. Households are perfectly mobile between communities and only differ by their exogenously determined endowment of the numeraire (income). Income ($y$) is thus independent of residential location choices. $F(y)$ and $f(y)$ are, respectively, the cumulative distribution function and the probability density function of income. The mass of households is normalised to one.

Because all houses in the system are homogenous and each family consumes one unit of
housing, this good is not an argument in the utility function representing households' preferences over different bundles in the economy. Therefore, preferences are defined over educational services received by the children (x) and household consumption of the numeraire (b).

**Assumption 1** Households have the same preferences represented by a utility function \( U(x,b) = u(x) + z(b) \). \( u(x) \) and \( z(b) \) are both increasing and twice continuously differentiable for all \((x,b) > 0\). \( u(x) \) is strictly quasi-concave, while \( z(b) \) is strictly concave.

Preferences are, therefore, continuous, strictly convex over and strictly monotonic. An important consequence of assumption 1 is that educational services and the numeraire are normal goods.

**Assumption 2**

\[
\lim_{x \to 0} u(x) = -\infty \quad \text{and} \quad \lim_{b \to 0} z(b) = -\infty
\]

Assumption 2 is for technical convenience. It ensures that any strictly positive combination of is strictly preferred to any bundle with one of the goods equal to zero.

2.4. Households’ decisión problem and timing

Each adult must adopt the following decisions: (i) choose the community in which to reside; (ii) decide to send her child to the local public school or to a private school somewhere; (iii) vote on the pair \((E, t)\) in the community in which the household resides; and, if her child attends a private school, (iv) allocate income between private education and numeraire consumption. Because households are atomistic, adults behave as price-takers. Consequently, when adopting all these decisions they take all community variables as given.

These decisions are made in two stages within a single period. In the first stage, households simultaneously choose communities and schools, taking into account their (correct) expectations over the equilibrium vector of public policies and housing prices \( e^* = (E_1, t_1, p_h^1, ..., E_J, t_J, p_h^J) \). In this stage, since the supply of housing is fixed, local housing markets clear. In the second one, once residence and schools decisions are committed, adults vote on their community education policy.

This sequence of decisions, can also be found in Nechyba (1999, 2002) and Eppe and Romano (2002). It is essential for solving the well known non single-peakedness problem that arises in models of public provision of education with opting-out (Stiglitz, 1974).
2.5. Definition of equilibrium

In this model, an equilibrium is a partition of households across communities and schools, an allocation \((x,b)\) across households and a vector of community policies and housing prices \(e^*=(E_1,t_1,p_h^1,...,E_J,t_J,p_h^J)\) satisfying:

1. **Rational choices**: for each household \(j\), the pair \((x_j,b_j)\) associated to their choice of community and school provides the maximum utility among the alternatives available in their choice set. This implies that no household wants to move to another community or to shift school.

2. **Housing market equilibrium**: housing demand equals housing (fixed) supply in every community.

3. **Majority voting equilibrium**: for all \(i=1,2,...,J\), the pair \((E_i,t_i)\) is majority-preferred by voters in community \(i\), given the partition of households across schools and the price of housing in the community. A pair \((E_i,t_i)\) is majority preferred in community \(i\) if the associated pair \((E,p_h(1+t))\) satisfies the government budget constraint (GBC), and if it is preferred by 50 percent or more of community \(i\) voters in a binary comparison against any other bundle satisfying the GBC.

3. WHO OPTS OUT OF PUBLIC EDUCATION?

3.1. Induced preferences

In order to clarify how households distribute themselves across communities and schools, we first obtain indirect utility functions corresponding to households choosing public and private education, respectively. From a household point of view, communities are characterised by the pair \((E,p_h(1+t))\), i.e. by the combination of expenditures per student (which ascertains the quality of the local public school) and the gross-of-tax price of housing (which determines the maximum feasible level of private consumption in the community. For this reason, such indirect utility functions are used to depict the indifference map in that space.

On the one hand, a household's decision-maker that sends their children to the local public school does not acquire any private education and, from strict monotonicity, devotes \(y-p_h(1+t)\) to consumption of the private composite commodity. The corresponding indirect utility function is:

\[
v(E, y - p) = u(E) + z(y - p_h(1 + t))
\]  

(1)

Let \(p\) be equal to \(p_h(1+t)\) and \(M(E,y-p)\) be the slope of indifference curves in this space. This slope is given by:
\[
\frac{dp}{dE} = M(E, y - p) = -\frac{v_E(E, y - p)}{v_p(E, y - p)} = \frac{u'(E)}{z'(y - p)} > 0
\]  

(2)

It is equal to the marginal benefit of education in terms of the numeraire. This implies that, in response to a marginal increase in \( E \), a household is willing to accept an increase in the gross-of-tax price of housing equal to the marginal benefit it obtains from education.

On the other hand, if the household chooses private education, a demand function for private education, \( x(y - p) \), must be obtained. The indirect utility function can then be written as:

\[
w(y - p) = u(x(y - p)) + z(y - p - x(y - p))
\]  

(3)

In this case, because the child does not attend the local public school, marginal benefit of public educational services is zero and, thus, indifference curves in (\( E, p \)) space are flat at each level of \( p \).

For a utility-maximizing household choosing between public and private schooling in a given community, the induced utility function is,

\[
V(E, y - p) = \max\{v(E, y - p), w(y - p)\}
\]  

(4)

The complete indifference map in (\( E, p \)) space is in figure 1. \( \hat{E}(y - p) \) is the locus of points at which the household is exactly indifferent between public and private schooling. It is possible to show that, for each pair (\( y - p \)), there is only one level of \( E \) at which this is satisfied. The indifference map sketched in figure 1 is analogue to that in Epple and Romano (1996). It shows that, given the gross-of-tax price of housing in the community, \( p \), a household with income \( y \) prefers private education for low enough levels of public provision \( (E < \hat{E}(y - p)) \), is exactly indifferent between the local public school and private schools for \( (E = \hat{E}(y - p)) \), and prefers public education for large enough amounts of public educational services \( (E > \hat{E}(y - p)) \). The flat part of indifference curves corresponds to the range in which the household prefers the private sector and is, thus, indifferent with respect to \( E \). The increasing part corresponds to the range in which the household uses a public school. In such range, an increase in \( p \) must be compensated with an increase in \( E \) in order to maintain indifference. For any indifference curve, the upper contour set is below it and the lower contour set is above it. Lemma 1 analyses the behaviour of \( \hat{E}(y - p) \).

**Lemma 1**  
\( \hat{E}(y - p) \) is everywhere increasing in \( y - p \).

**Proof**  
Differentiate \( v(\hat{E}(y - p), y - p) = w(y - p) \) with respect to \( (y - p) \);
\[ d(y - p)\left(u'(\hat{E}(y - p)\hat{E}'(y - p) + z'(y - p))\right) = d(y - p)(z'(y - p - x(y - p)) + \hat{E}'(y - p) - \hat{E}'(y - p)) \]

Solve for \( \hat{E}'(y - p) \) to obtain:

\[ \hat{E}'(y - p) = \frac{z'(y - p - x(y - p)) - z'(y - p)}{u'(\hat{E}(y - p))} \]

(5)

Given that assumption 2 assures a strictly positive demand for private education when private schooling is chosen, the latter inequality is guaranteed by concavity of \( z(\cdot) \). Q.E.D.

From lemma 1 it is immediate to establish:

**Corollary 1**  **Within-communities perfect income stratification across schools.** If for any \((E_j, p_j)\) a household with income \( y' \) residing in community \( j \) is indifferent between private and public education, then all households with income \( y > y' \) (\( y < y' \)) residing in that community strictly prefer the private (public) option.

**Proof**  Let \( y \) be such that, given \((E_j, p_j)\), households with income equal to \( y \) in community \( j \) are indifferent between private and public education. This entails that \( E = \hat{E}(y' - p) \). Lemma 1 proves that \( \hat{E}(y - p) \) rises monotonically with income. Hence, all households with income \( y > y' \), satisfy \( E < \hat{E}(y - p) \), strictly preferring the private alternative. Similarly, for all households with income \( y < y' \), \( E > \hat{E}(y - p) \), and, therefore, they strictly prefer the public option. Q.E.D.

Corollary 1 shows that, in equilibrium, mixed communities are characterised by perfect income stratification across schools. The poorest households send their children to the local public school and the richest ones opt out of the public system and acquire private education. This result is a standard prediction in the literature (e.g. Epple and Romano, 1996 and Bearse et al., 2001).

### 3.2. Rational residential choices

We now turn to the analysis of optimal residential choices of households. In doing so, we take as given the choice among public and private education. For households choosing private education, it is shown that the rational residential location is the community with the lowest gross-of-tax price of
housing. For households sending their children to one of the local public schools, we provide some conditions that characterise rational residential choices. For expositional convenience and to avoid uninteresting cases, we adopt the following realistic assumption.

**Assumption 3** All communities provide a positive amount of public educational services.

Assumption 3 restricts attention to empirically relevant equilibria in which all communities are inhabited by (some) households sending their youths to the local public school.

The analysis proceeds as follows. Lemma 2 establishes an ordering of communities which must be satisfied by any vector of public policies and housing prices to be a candidate for equilibrium. Communities are then numbered according to this ordering. Using such ranking and taking school choices as given, propositions 1 and 2 establish conditions that must be satisfied by residential choices to be rational.

**Lemma 2** In equilibrium, for every pair of communities $i$ and $j$: $E_j > E_i \iff p_j > p_i$ and $E_j = E_i \iff p_j = p_i$.

**Proof** An allocation with $E_j > E_i$ and $p_j \leq p_i$ cannot be an equilibrium because in that case $u(E_j) > u(E_i)$, $z(y-p_j) \geq z(y-p_i)$ and, therefore, $v(E_j, y-p_j) > v(E_i, y-p_i)$ for all $y$. Consequently, all households choosing public education in community $i$ would want to move to community $j$, which is incompatible with our definition of equilibrium. A similar argument serves to prove the second part of the lemma. Q.E.D.

Housing prices, therefore, serve as screening mechanisms and differences in $E$ are capitalised to some extent into housing prices. Those communities with higher provision levels also have higher gross-of-tax housing prices and those with identical level of provision have equal gross-of-tax housing prices. Henceforth, we shall assume that all communities have different gross-of-tax housing prices. All results below extend to the case in which some communities have the same price just by considering them a community group which is treated as a single community. Let communities be numbered such that $(E_i, p_i) < (E_{i+1}, p_{i+1})$ for all $i=1, 2, ..., J-1$.

**Proposition 1** In equilibrium, all households using private schools reside in the
community with the lowest gross-of-tax price of housing (community 1).

Proof. Strong monotonicity of preferences makes \( w(y-p) \) to be everywhere increasing in \( y-p \). Because \( p_1 < p_j \) for all \( j=2,...,J \), \( y-p_1 > y-p_j \) and \( w(y-p_1) > w(y-p_j) \) for all \( j=2,...,J \). Q.E.D.

Proposition 1 states that, in equilibrium, households who opt out of public education reside in the community with the lowest gross-of-tax housing price. This result is easily deduced from the indifference map in figure 1 and occurs because these households only care about the level of private consumption in each community.

Proposition 2 In equilibrium, (i) Perfect income stratification across public schools. Households using public schools are perfectly stratified by income across communities. (ii) Ascending bundles. Let \( \hat{y}_i^u \) be the income of the richest household in community \( i \) consuming public education. All communities satisfy the following ascending bundles condition. If \( \hat{y}_j^u > \hat{y}_i^u \) \( \Rightarrow \) \( (E_j, p_j) \gg (E_i, p_i) \).

Proof. (i) Slope of indifference curves corresponding to \( v(E, y-p) \) in \( (E, p) \) space, \( M(E, y-p) \), increases monotonically with \( y \):

\[
\frac{\partial M(E, y-p)}{\partial y} = -\frac{u'(E)z'(y-p)}{z'(y-p)^2} > 0
\]

(6)

An important consequence of this slope rising in income (SRI) condition is the following single-crossing property: the indifference curve of a household with income \( y \) crosses the indifference curve of any other household with different income at most once, and the indifference curve of the wealthier of any two households always cuts that of the poorer from below. The single-crossing property, in turn, implies the following preference ordering, proved in Epple et al. (1993), lemma 1: given \( (E_i, p_i) \ll (E_j, p_j) \),

\[
v(E_i, y-p) \geq v(E_j, y-p) \Rightarrow v(E_i, y'-p_i) > v(E_j, y'-p_j), \quad \forall y' < y \quad (7a)
\]

\[
v(E_i, y-p) \leq v(E_j, y-p) \Rightarrow v(E_i, y'-p_i) < v(E_j, y'-p_j), \quad \forall y' > y \quad (7b)
\]

(7a) and (7b) entail that, in equilibrium, a community cannot be inhabited by households in the public sector from different income intervals.

(ii) By contradiction. Suppose that in equilibrium \( (E_j, p_j) \gg (E_i, p_i) \) and \( \hat{y}_j^u < \hat{y}_i^u \). In that case, the following conditions must hold (a) \( v(E_i, \hat{y}_i^u - p_i) \geq v(E_j, \hat{y}_j^u - p_j) \), and (b)
\[ v(E_j, \hat{y}_j^n - p_j) \geq v(E_i, \hat{y}_i^n - p_i) \]. From (7a), however, we know that,
\[ v(E_i, \hat{y}_i^n - p_i) \geq v(E_j, \hat{y}_j^n - p_j) \Rightarrow v(E_i, y_i' - p_i) > v(E_j, y_j' - p_j); \forall y_i' < \hat{y}_i^n \]

And, therefore, (b) cannot hold if \( \hat{y}_j^n < \hat{y}_i^n \). Q.E.D.

### 3.3. Income stratification across public and private education

We now turn to the question of which households use public schools and which decide to opt out of the public system and acquire private education. Without loss of generality, we set \( J=2 \) for all the analysis below. Define \( \hat{E}_2(y, p_1, p_2) \) as community 2 level of provision at which households with income \( y \) are just indifferent between private education at community 1 and public education in community 2. \( \hat{E}_2(y, p_1, p_2) \) is a function, i.e. for each \( (y, p_1, p_2) \) there exists a unique \( \hat{E}_2 \) for which \( v(\hat{E}_2, y - p_2) = w(y - p_1) \). In \((E,p)\) space \( \hat{E}_2(y, p_1, p_2) \) coincides with the indifference curve corresponding to \( v(E, y - p) \) and to a utility level equal to \( w(y - p_1) \) (see figure 2). Given the actual level of provision in community 2, \( E_2 \), households with income \( y \) such that \( \hat{E}_2(y, p_1, p_2) > E_2 \) strictly prefer private education at community 1, while households with income \( y \) such that \( \hat{E}_2(y, p_1, p_2) < E_2 \) strictly prefer living in community 2 and sending their children to the local public school there. Lemma 3 contains results regarding the behaviour of this function.

**Lemma 3** \( \hat{E}_2(y, p_1, p_2) \) is first decreasing in income for levels of income satisfying \( p_2 - p_1 > x(y - p_1) \), reaches a minimum at \( y \) such that \( p_2 - p_1 = x(y - p_1) \) and then increases with income for levels such that \( p_2 - p_1 < x(y - p_1) \).

**Proof** \( \hat{E}_2(y, p_1, p_2) \) is implicitly defined by
\[ v(\hat{E}_2(y, p_1, p_2), y - p_2) = \frac{\partial \hat{E}_2(\cdot)}{\partial y} w(y - p_1) \]. Differentiate this expression with respect to \( y \) and solve for \( \frac{\partial \hat{E}_2(\cdot)}{\partial y} \) to obtain:
\[ \frac{\partial \hat{E}_2(\cdot)}{\partial y} = \frac{z'(y - p_1 - x(y - p_1)) - z'(y - p_2)}{u'(\hat{E}_2(\cdot))} \] (8)

Again we use assumption 2, which guarantees an interior solution for the utility maximisation problem of households using private schools. Given strict concavity of \( z(\cdot) \), this
derivative will be negative (positive) if \( y - p > x(y - p_1) > y - p_2 \) (i.e., if \( p_2 > p > x(y - p_1) \)). Finally note that \( x(y - p_1) \) rises monotonically with income. Q.E.D.

Lemma 3 implies that if households indifferent between "private education-community 1" and "public education-community 2" have a demand for private education in community 1 smaller than the difference in gross-of-tax housing prices between community 2 and community 1 (i.e., if for these households consumption of the hicksian commodity is larger in the former alternative), then richer (poorer) households strictly prefer the latter (former) of both alternatives. Note that, if this occurs, it seems possible to find equilibria in which households from intermediate income intervals opt out of the public system, while poorer and richer households continue using public schools. This is confirmed by the example of equilibrium in proposition 3.

Consider an economy corresponding to the model in section 2. This economy is composed of two communities, 1 and 2. Households preferences are captured by the utility function, borrowed from Bearse et al. (2001):

\[
U(x,b) = \frac{1}{1-\sigma} \left[ b^{1-\sigma} + \delta x^{1-\sigma} \right] \sigma, \delta > 0
\]

This utility function is separable in \((b,x)\) and strictly concave for \(\sigma, \delta > 0\). It violates assumption 2 but this is inconsequential for the example in proposition 3. Parameters of the utility function are set at levels in Bearse et al. (2001): \(\sigma = 2.23\) and \(\delta = 0.0032\). The income cumulative distribution function is given by \(F(y)\) with lower and upper bounds \(\underline{y}\) and \(\overline{y}\).

**Proposition 3**  Example 1 below is an equilibrium of the above described economy.

**Proof.** See Martínez-Mora (2003).

**Example 1**  Communities capacity: \(N_1=0.75; N_2=0.25\).

Income cumulative distribution function (figure 3):

\[
F(y) = 0.2232 + 3.6261 \cdot 10^{-2} y - 1.2669 \cdot 10^{-3} y^2 + 3.0696 \cdot 10^{-5} y^3 - 3.8289 \cdot 10^{-7} y^4 + 2.2992 \cdot 10^{-9} y^5 - 5.2834 \cdot 10^{-12} y^6; \underline{y} = 8; \overline{y} = 125
\]

Vector of public policies and housing prices: \(e^*=(E_1,t_1,p_1^h,E_2,t_2,p_2^h)=(2,0.27,5.9,6.2,0.89,7)\)

Median voters: \((\overline{y}_1, \overline{y}_2) = (23.37, 94.70)\)

Partition of households across communities and schools: households with income \(y \in [89.88]\) (with a mass equal to 0.6) choose public education-community 1; households with income
y \in [59.88, 71.96) \) (with a mass equal to 0.1) choose private education-community 1; households with income \( y \in [71.96, 117.47) \) (with a mass equal to 0.25) choose public education-community 2; households with income \( y \in [117.47, 125) \) (with a mass equal to 0.05) choose private education-community 1.

4. DISCUSSION AND CONCLUDING REMARKS

Households that choose private schooling are those that do not find a community with the mix of price of housing, tax rate and per-student expenditures which leads to the combination of numeraire consumption and school quality within their (non-convex) feasible set providing the maximum utility. For these households, then, it is worth spending some extra money in order to acquire private educational services. In the single jurisdiction case, these households are always the rich ones, whose demand for education is larger than the common provision level. As example 1 above demonstrates, in a multi-community setting this result does not necessarily holds.

Figure 4 shows, for this example, the level of education received by households from each level of income, given their choice of community and school. Households from an intermediate income interval are not satisfied by the mix of public education and housing prices offered by communities 1 and 2. Consequently, they exit the public system and send their youths to private schools. These schools are of higher quality than the public school in community 1 but of lower quality than the public school in community 2. That is to say, for these households, \( E_1 < x_1(\cdot) < E_2 \), as can be checked in figure 4. On the other hand, their level of numeraire consumption is larger than that feasible in community 2 and obviously smaller than the amount they could purchase were they using the local public school in community 1.

The reason why they choose private schooling is that they prefer an intermediate combination of school quality and numeraire consumption to the alternatives offered by public education in communities 1 and 2. Clearly, the convexity property satisfied by the preference relation in the model is key for this result to hold. For these households, the quality of the local public school of community 1 is too low given the level of disposable income they would have if they lived there \( (y-p_1) \). But also, because the price of housing is very high in community 2, the level of disposable income they would have if they instead chose to live in community 2 \( (y-p_2) \) is too low, given the quality of public education in that community.

Another key ingredient for the existence of equilibria of this type are housing markets. Public school quality differentials are capitalised to some extent into housing prices. And it is the
existence of a large difference in (gross-of-tax) housing prices among communities what makes these households to have too high a level of disposable income in community 1 to choose the local public school there and, at the same time, too low a level of disposable income in community 2 to prefer the local public school there.

An important question for further research is under which circumstances this type of equilibrium is more likely to arise. The analysis in this paper suggests that the more polarised the income distribution and the larger the income elasticity of demand for educational services the higher this probability will be. Intuitively, this occurs because differences in public education quality and, consequently, in housing prices among communities will be larger under such circumstances. Such probability also rises with the elasticity of substitution among numeraire and educational services. The reason is that this makes households to have a stronger preference for intermediate combinations of numeraire and educational services. The number of communities can also be a relevant variable. In general, the larger the number of communities, the larger the menu of public alternatives and the smaller the number of households (from whatever income interval) choosing private education.

Last but not least, it should be stressed that this paper highlights, once again, how profoundly housing markets affect the market for education. In our model, a simple specification of housing markets serves to show how difficult it is to predict the way in which households and their children sort into communities and schools, and the key role housing markets play in this process. From our point of view, models that allow to capture the impact of housing markets into the market for education will be an important instrument in future research, mainly, on topics related to school choice and competition among private and public educational institutions.

5. REFERENCES


